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SLC/LEP CONSTRAINTS ON UNIFIED MODELS

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ABSTRACT

We examine the potential of constraining possible nondecoupling effects of heavy neutrinos and Higgs bosons at LEP and SLC that may show up in the nonoblique part of the Zl_il_j couplings. We analyze this type of new-physics interactions within the context of low-energy scenarios motivated by unified theories, such as the Standard Model (SM) with neutral isosinglets, the left-right symmetric model, and the minimal supersymmetric SM. Our analysis comprises a complete set of physical quantities based on the nonobservation of flavour-violating Z-boson decays, lepton universality in the decays $Z \to l\bar{l}$, and universality of lepton asymmetries at the Z peak. It is found that these quantities form a set of complementary observables and may hence constrain the parameter space of the theories. Non-SM contributions of new-physics interactions to $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ are briefly discussed within these models.

1. Introduction

In view of the recent discrepancy of about 2σ standard deviations between the leptonic asymmetry $\mathcal{A}_e$ measured at the Large Electron Positron Collider (LEP) [1] and the left-right asymmetry $\mathcal{A}_{LR}$ at the Stanford Linear Collider (SLC),[2] one may have to face the fact that the minimal Standard Model (SM) may not be the underlying theory of nature. [3] If our understanding of nature is due to some unified theory, such as supersymmetry (SUSY), grand unified theories (GUTs), superstrings, etc., it is then important to know the size of new-physics effects expected to come from such theories at LEP and SLC. Analyzing electroweak oblique parameters has become a common strategy to test the viability of models beyond the SM, especially when new physics couples predominantly to W and Z bosons.[4,5] However, one has to explore additional observables that could be more sensitive to other sectors of the SM.

In Section 2, we will therefore focus our discussion on observables exhibiting lepton-universality and lepton-flavour violation via the Zl_1l_2 couplings and describe the experimental situation at LEP/SLC. Then, we will analyze new-physics interactions in the leptonic sector within the context of low-energy extensions of the SM that could be motivated by SUSY-GUTs, such as SUSY-$SO(10)$. Such theories [6] have received much attention, since the the electroweak mixing angle, $\sin^2\theta_w(\equiv s_w^2)$,

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predicted is in excellent agreement with its value measured experimentally. Also, the supersymmetric nature of a SUSY-GUT model prefers higher unification-point values than usual GUTs, which makes proton practically "stable" with a lifetime of order $10^{36} - 10^{38}$ years. Obviously, the low-energy limit of a SUSY-GUT scenario depends crucially on the field content and the details of the breaking mechanism from the unification scale down to the electroweak one.[7] In particular, we shall discuss the phenomenological implications of three representative extensions of the SM for the leptonic sector, which could also be the low-energy limit of certain SUSY-GUTs. Thus, Sections 3, 4, and 5 deal correspondingly with the SM with left-handed and/or right-handed neutral isosinglets, the left-right symmetric model (LRSM), and the minimal SUSY-SM. In Section 6, we will briefly discuss predictions obtained for R_b within these models. We draw our conclusions in Section 7.

2. The Zl_1l_2 vertex

Here, we define more precisely the framework of our calculations. In the limit of vanishing charged lepton masses, the decay amplitude for $Z \to l_1\bar{l}_2$ can generally be parametrized as

$$\mathcal{T}_l \;=\; \frac{ig_w}{2c_w}\,\varepsilon^\mu_Z\,\bar{u}_{l_1}\gamma_\mu[g^{l_1l_2}_L \mathrm{P}_L \;+\; g^{l_1l_2}_R \mathrm{P}_R]v_{l_2}, \tag{1}$$

where g_w is the usual electroweak coupling constant, ε^μ_Z is the Z-boson polarization vector, u (v) is the Dirac spinor of the charged lepton l_1 (l_2), $\mathrm{P}_L(\mathrm{P}_R) = (1-(+)\gamma_5)/2$, and $c^2_w = 1 - s^2_w = M^2_W/M^2_Z$. In Eq. (1), we have defined

$$g^{l_1l_2}_{L,R} = g^l_{L,R} + \delta g^{l_1l_2}_{L,R}, \qquad g^l_L = \sqrt{\rho_l}(1-2\bar{s}^2_w), \qquad g^l_R = -2\sqrt{\rho_l}\bar{s}^2_w, \tag{2}$$

where ρ_l, $\bar{s}_w$, $\delta g^{ll}_{L,R}$ $(\equiv \delta g^l_{L,R})$ are obtained beyond the Born approximation and are renormalization-scheme dependent. In particular, ρ_l, $\bar{s}_w$ introduce universal oblique corrections,[4,5] whereas $\delta g^{l_1l_2}_{L,R}$ represent flavour-dependent corrections. It is obvious that an analogous expression is valid for the decay $Z \to b\bar{b}$, as soon as b-quark mass effects can be neglected.

To facilitate our presentation, we reexpress the flavour-dependent electroweak corrections in terms of the loop functions $\Gamma^L_{l_1l_2}$ and $\Gamma^R_{l_1l_2}$ as follows:

$$\delta g^{l_1l_2}_L \;=\; \frac{\alpha_w}{2\pi}\,\Gamma^L_{l_1l_2}, \qquad \delta g^{l_1l_2}_R \;=\; \frac{\alpha_w}{2\pi}\,\Gamma^R_{l_1l_2},$$

with $\alpha_w = g^2_w/4\pi$. The nonoblique loop functions $\Gamma^L_{l_1l_2}$ and $\Gamma^R_{l_1l_2}$ depend on whether the underlying theory is of V–A or V+A nature. Then, the branching ratio for possible decays of the Z boson into two different charged leptons is given by

$$B(Z\to \bar{l}_1 l_2 + l_1\bar{l}_2) \;=\; \frac{\alpha^3_w}{48\pi c^2_w}\,\frac{M_Z}{\Gamma_Z}\Big[|\Gamma^L_{l_1l_2}|^2 + |\Gamma^R_{l_1l_2}|^2\Big]. \tag{3}$$

This kind of non-SM decays are constrained by LEP results to be, *e.g.*, $B(Z \to e\tau) \lesssim 10^{-5}$. [8]

Another observable that has been analyzed recently [9] is the universality-breaking parameter $U_{br}^{(l_1 l_2)}$. To leading order of perturbation theory, $U_{br}^{(l_1 l_2)}$ is given by

$$\begin{aligned} U_{br}^{(l_1 l_2)} &= \frac{\Gamma(Z \to l_1 \bar{l}_1) \ - \ \Gamma(Z \to l_2 \bar{l}_2)}{\Gamma(Z \to l_1 \bar{l}_1) \ + \ \Gamma(Z \to l_2 \bar{l}_2)} \ - \ U_{br}^{(l_1 l_2)}(\mathrm{PS}) \\ &= \frac{g_L^l(\delta g_L^{l_1} - \delta g_L^{l_2}) \ + \ g_R^l(\delta g_R^{l_1} - \delta g_R^{l_2})}{g_L^{l2} \ + \ g_R^{l2}} \\ &= U_{br}^{(l_1 l_2)}(\mathrm{L}) \ + \ U_{br}^{(l_1 l_2)}(\mathrm{R}). \end{aligned} \tag{4}$$

In Eq. (4), $U_{br}^{(l_1 l_2)}(\mathrm{PS})$ characterizes known phase-space corrections coming from the nonzero masses of the charged leptons l_1 and l_2 that can always be subtracted, and

$$U_{br}^{(l_1 l_2)}(\mathrm{L}) = \frac{g_L^l(\delta g_L^{l_1} - \delta g_L^{l_2})}{(g_L^{l2} + g_R^{l2})} = \frac{\alpha_w}{2\pi} \frac{g_L^l}{g_L^{l2} + g_R^{l2}} \Re e(\Gamma_{l_1 l_1}^L - \Gamma_{l_2 l_2}^L), \tag{5}$$

$$U_{br}^{(l_1 l_2)}(\mathrm{R}) = \frac{g_R^l(\delta g_R^{l_1} - \delta g_R^{l_2})}{g_L^{l2} + g_R^{l2}} = \frac{\alpha_w}{2\pi} \frac{g_R^l}{g_L^{l2} + g_R^{l2}} \Re e(\Gamma_{l_1 l_1}^R - \Gamma_{l_2 l_2}^R). \tag{6}$$

Lepton asymmetries — or equivalently forward-backward asymmetries — can also be sensitive to new physics. Here, we will be interested in experiments at LEP/SLC that measure the observable

$$\begin{aligned} \mathcal{A}_l &= \frac{\Gamma(Z \to l_L \bar{l}) \ - \ \Gamma(Z \to l_R \bar{l})}{\Gamma(Z \to l\bar{l})} = \frac{g_L^{ll2} \ - \ g_R^{ll2}}{g_L^{ll2} \ + \ g_R^{ll2}} \\ &= \frac{g_L^{l2} - g_R^{l2} + 2(g_L^l \delta g_L^l - g_R \delta g_R^l)}{g_L^{l2} + g_R^{l2} + 2(g_L^l \delta g_L^l + g_R^l \delta g_R^l)}. \end{aligned} \tag{7}$$

In particular, we use the nonuniversality parameter of lepton asymmetries [10]

$$\Delta\mathcal{A}_{l_1 l_2} = \frac{\mathcal{A}_{l_1} \ - \ \mathcal{A}_{l_2}}{\mathcal{A}_{l_1} \ + \ \mathcal{A}_{l_2}} = \frac{1}{\mathcal{A}_l^{(SM)}} \Big(U_{br}^{(l_1 l_2)}(\mathrm{L}) \ - \ U_{br}^{(l_1 l_2)}(\mathrm{R}) \Big) \ - \ U_{br}^{(l_1 l_2)}, \tag{8}$$

where $\mathcal{A}_l^{(SM)}$ may be given by the SM value. We also emphasize that $U_{br}^{(l_1 l_2)} = 0$ does *not necessarily* imply $\Delta\mathcal{A}_{l_1 l_2} = 0$. For instance, LRSMs can naturally generate situations, in which $U_{br}(\mathrm{L}) \simeq -U_{br}(\mathrm{R})$ while $\Delta\mathcal{A}_{l_1 l_2}$ becomes sizeable. Moreover, the physical quantities $U_{br}^{(l_1 l_2)}$ and $\Delta\mathcal{A}_{l_1 l_2}$ do not depend explicitly on universal electroweak oblique parameters.

A recent combined analysis of the LEP/SLC results regarding lepton universality at the Z peak gives [1,2]

$$\begin{aligned} |U_{br}^{(ll')}| &< 5.\ 10^{-3} \qquad (\mathrm{SM}: 0), \\ \mathcal{A}_\tau(\mathcal{P}_\tau) &= 0.143 \pm 0.010 \qquad (\mathrm{SM}: 0.143), \end{aligned}$$

$$
\begin{aligned}
\mathcal{A}_e(\mathcal{P}_\tau) &= 0.135 \pm 0.011, \\
\mathcal{A}_{FB}^{(0,l)} &= 0.0170 \pm 0.0016 \qquad (\mathrm{SM}: 0.0153), \\
\mathcal{A}_{LR}(SLC) &= 0.1637 \pm 0.0075,
\end{aligned} \tag{9}
$$

where theoretical predictions obtained in the SM are quoted in the parentheses. Note that $\mathcal{A}_e$ from τ polarization is 2σ away from the left-right asymmetry, $\mathcal{A}_{LR}$, measured at SLC. From Eq. (9), one can deduce $\Delta\mathcal{A}_{\tau e} \simeq -10\%$ when comparing measurements at LEP and SLC. However, if one assumes that the measurement of $\mathcal{A}_{LR}$ is correct, then one could interpret the experimental sensitivity for $\mathcal{A}_{LR}$ as a stronger upper bound on new physics with $|\Delta\mathcal{A}_{\tau e}| < 4\%$. Furthermore, ongoing SLC experiments are measuring the observable

$$
A_{LR}^{FB}(f) \;=\; \frac{\Delta\sigma(e_L^- e^+ \to f\bar{f})_{FB} - \Delta\sigma(e_R^- e^+ \to f\bar{f})_{FB}}{\Delta\sigma(e_L^- e^+ \to f\bar{f})_{FB} + \Delta\sigma(e_R^- e^+ \to f\bar{f})_{FB}} = \frac{3}{4}\mathcal{P}_e\mathcal{A}_f, \tag{10}
$$

The forward-backward left-right asymmetry for individual flavours will be an interesting alternative of testing lepton universality in the SM in the near future.

3. The SM with neutral isosinglets

Here, we will adopt the conventions and the model of Ref. 11, for the charged- and neutral-current interactions. The model extends the SM by more than one neutral isosinglets, which allows the presence of large Dirac components in the general Majorana neutrino mass matrix. The couplings WlN_i and ZN_iN_j to charged leptons l and heavy Majorana neutrinos N_i are mediated by the mixings B_{lN_i} and $C_{N_iN_j}$, respectively. For a model with two-right handed neutrinos, for example, we have [12,13]

$$
B_{lN_1} \;=\; \frac{\rho^{1/4} s_L^{\nu_l}}{\sqrt{1+\rho^{1/2}}}\ , \qquad B_{lN_2} \;=\; \frac{i s_L^{\nu_l}}{\sqrt{1+\rho^{1/2}}}\ , \tag{11}
$$

where $\rho = m_{N_2}^2/m_{N_1}^2$ is the square of the mass ratio of the two heavy Majorana neutrinos N_1 and N_2 present in such a model. The lepton-flavour mixings $s_L^{\nu_l}$ are defined as: [14] $(s_L^{\nu_l})^2 \equiv \sum_{j=1}^{2} |B_{lN_j}|^2$. Furthermore, the mixings $C_{N_iN_j}$ can be obtained by $\sum_{l=1}^{3} B_{lN_i}^* B_{lN_j} = C_{N_iN_j}$. The mixing angles $(s_L^{\nu_i})^2$ are directly constrained by low-energy and other LEP data.[15] Although some of the constraints could be model-dependent, we use the conservative upper limits: [15] $(s_L^{\nu_e})^2$, $(s_L^{\nu_\mu})^2 < 0.01$, and $(s_L^{\nu_\tau})^2 < 0.06$.

Flavour-changing neutral current decays (FCNC) of the Z boson into two different charged leptons were found to receive enhancements due to heavy-neutrino nondecoupling effects.[16,12,13 a]

[a] In general *three-generation* Majorana-neutrino mass models, nondecoupling effects of heavy neutrinos due to large Dirac components, which result obviously from the spontaneous break-down of the $SU(2)_L$ gauge symmetry, have originally been discussed by the author in relation with FCNC Higgs boson decays, $H \to l\bar{l}'$.[17]

To leading order of heavy neutrino masses, the branching ratio of this kind of decays is given by

$$B(Z \to e^-\tau^+ + e^+\tau^-) \;=\; \frac{a_w^3}{768\pi^2 c_w^3}\,\frac{M_W}{\Gamma_Z}\,\frac{m_N^4}{M_W^4}\,(s_L^{\nu_e})^2(s_L^{\nu_\tau})^2\Big[\sum_i (s_L^{\nu_i})^2\Big]^2, \qquad (12)$$

where Γ_Z is the total width of the Z boson. An optimistic theoretical prediction of these decay modes gives $B(Z \to e\tau) < 10^{-6}$, which should be compared with the present experimental sensitivity of order 10^{-5}. On the other hand, taking $\lambda_{N_1} = m_{N_1}^2/M_W^2 \gg 1$ and $\rho = m_{N_2}^2/m_{N_1}^2 \geq 1$ into account,[9] the universality-breaking parameter U_{br} can compactly be given by

$$\begin{aligned} U_{br}^{(ll')} \;=\; U_{br}^{(ll')}(\mathrm{L}) \;=\; & -\frac{\alpha_w}{8\pi}\,\frac{g_L}{g_L^2+g_R^2}\Big((s_L^{\nu_l})^2 - (s_L^{\nu_{l'}})^2\Big)\Bigg[3\ln\lambda_{N_1} \\ & + \sum_{i=1}^{n_G}(s_L^{\nu_i})^2\,\frac{\lambda_{N_1}}{(1+\rho^{\frac{1}{2}})^2}\Bigg(3\rho + \frac{\rho - 4\rho^{\frac{3}{2}} + \rho^2}{2(1-\rho)}\ln\rho\Bigg)\Bigg]. \qquad (13) \end{aligned}$$

Another attractive low-energy scenario is an extension of the SM inspired by certain GUTs [18] and superstring theories,[19] in which left-handed neutral singlets in addition to the right-handed neutrinos are present. In this scenario, the light neutrinos are strictly massless to all orders of perturbation theory,[18] when $\Delta L = 2$ operators are absent from the Yukawa sector. The minimal case with one left-handed and one right-handed chiral singlets can effectively be recovered by the SM with two right-handed neutrinos when taking the degenerate mass limit for the two heavy Majorana neutrinos in Eq. (13). In Table 1, we present numerical results for both scenarios discussed above by assuming $m_{N_1} \simeq m_{N_2} = m_N$. The present experimental upper bound on $U_{br}^{(ll')}$ is $|U_{br}^{(ll')}| < 5.\ 10^{-3}$, [8] which automatically sets an upper limit on

Table 1. $U_{br}^{\tau e}(\mathrm{L})$, $U_{br}^{\tau e}(\mathrm{R})$, and $\Delta\mathcal{A}_{\tau e}$ in models discussed in Sects. 3 (i), 4 (ii), and 5 (iii).

Gauge	Models	$U_{br}^{\tau e}(\mathrm{L})$	$U_{br}^{\tau e}(\mathrm{R})$	$\Delta\mathcal{A}_{\tau e}$
(i) $(s_L^{\nu_\tau})^2$	m_N [TeV]			
0.035	4.0	$-4.0\ 10^{-3}$	0	$-2.4\ 10^{-2}$
0.020	4.0	$-2.0\ 10^{-3}$	0	$-1.2\ 10^{-2}$
(ii) M_R [TeV]	M_h [TeV]			
0.4	5	$-1.\ 10^{-2}$	$7.7\ 10^{-3}$	-0.13
0.4	50	$-1.\ 10^{-2}$	$1.5\ 10^{-2}$	-0.18
1.0	5	$-1.\ 10^{-2}$	$1.2\ 10^{-3}$	-0.16
1.0	100	$-1.\ 10^{-2}$	$3.2\ 10^{-3}$	-0.10
(iii) $\theta_L = 0$	$m_{\tilde{l}}$ [GeV]			
$\theta_R = \frac{\pi}{2}$	45	$1.1\ 10^{-4}$	$-6.4\ 10^{-5}$	$1.2\ 10^{-3}$
$m_{\tilde{l}_L} = m_{\tilde{l}_R}$	60	$5.5\ 10^{-5}$	$-3.1\ 10^{-5}$	$6.1\ 10^{-4}$
$= m_{\tilde{l}}$	100	$2.0\ 10^{-5}$	$-1.1\ 10^{-5}$	$2.2\ 10^{-4}$

$|\Delta\mathcal{A}_{ll'}| \lesssim 3\%$ since $U_{br}^{ll'}(\mathrm{R}) = 0$.

Fig. 1. Feynman diagrams inducing a right-handed non-universal $Zl\bar{l}$ coupling in the LRSM.

4. The LRSM

This model is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. We have worked out the realistic case (d) in Ref. 20, in which the vacuum expectation values of the left-handed Higgs triplet Δ_L and that of ϕ_2^0 in the Higgs bi-doublet vanish. In the LRSM, FCNC Z boson decays into two different charged leptons are calculated recently [21] and found to be of comparable order with those of the SM with neutral isosinglets. In addition, LRSMs can naturally give rise to both a left-handed and a right-handed non-universal $Zl\bar{l}$ coupling.[10,21] The expression for $U_{br}^{(ll')}(\mathrm{L})$ equals the one given in Eq. (6), while $U_{br}^{(ll')}(\mathrm{R})$ can be obtained by calculating the Feynman graphs shown in Fig. 1. In the limit where the charged gauge bosons $W_R^\pm$ and the charged Higgs bosons $h^\pm$ are much heavier than the Z boson, the dominant nondecoupling heavy neutrino and Higgs-scalar contributions to $U_{br}^{(ll')}(\mathrm{R})$ can be cast into the form [10]

$$\begin{aligned} U_{br}^{(ll')}(\mathrm{R}) \;&=\; \frac{\alpha_w}{8\pi}\,\frac{g_R}{g_L^2+g_R^2}\Big(B^R_{lN_i}B^{R*}_{lN_j} - B^R_{l'N_i}B^{R*}_{l'N_j}\Big)\sqrt{\lambda_{N_i}\lambda_{N_j}} \\ &\quad\times\Big[\,\delta_{ij}F_1 \;+\; C^L_{N_iN_j}F_2 \;+\; C^{L*}_{N_iN_j}F_3\Big], \end{aligned} \tag{14}$$

Fig. 2. Graphs contributing to a nonoblique $Zl\bar{l}$ coupling in the SUSY-SM.

where F_1, F_2, and F_3 are form factors given by

$$
\begin{aligned}
F_1 &= 4s_\beta^2[C_0(\lambda_R,\lambda_R,\lambda_{N_i}) \;-\; C_0(\lambda_R,\lambda_h,\lambda_{N_i})], && (15)\\
F_2 &= 2[C_0(\lambda_R,\lambda_h,\lambda_{N_i}) \;+\; C_0(\lambda_R,\lambda_h,\lambda_{N_j}) \;-\; C_0(\lambda_{N_i},\lambda_{N_j},\lambda_R)] \\
&\quad + s_\beta^2[C_{24}(\lambda_h,\lambda_h,\lambda_{N_i}) \;+\; C_{24}(\lambda_h,\lambda_h,\lambda_{N_j}) \;-\; C_{24}(\lambda_R,\lambda_h,\lambda_{N_i}) \\
&\quad - C_{24}(\lambda_R,\lambda_h,\lambda_{N_j}) \;+\; C_{24}(\lambda_{N_i},\lambda_{N_j},\lambda_R) \;-\; C_{24}(\lambda_{N_i},\lambda_{N_j},\lambda_h)] \\
&\quad + \frac{s_\beta^2}{c_\beta^2}\,[C_{24}(\lambda_{N_i},\lambda_{N_j},\lambda_h) \;-\; C_{24}(0,\lambda_{N_j},\lambda_h) \;-\; C_{24}(\lambda_{N_i},0,\lambda_h) \\
&\quad + C_{24}(0,0,\lambda_h)], && (16)\\
F_3 &= -\frac{2}{\sqrt{\lambda_{N_i}\lambda_{N_j}}}\,[C_{24}(\lambda_{N_i},\lambda_{N_j},\lambda_R) \;-\; C_{24}(0,\lambda_{N_j},\lambda_R) \;-\; C_{24}(\lambda_{N_i},0,\lambda_R) \\
&\quad + C_{24}(0,0,\lambda_R)] \;+\; s_\beta^2\,\sqrt{\lambda_{N_i}\lambda_{N_j}}\,C_0(\lambda_{N_i},\lambda_{N_j},\lambda_R), && (17)
\end{aligned}
$$

with $\lambda_R = 1/s_\beta^2 = M_R^2/M_W^2$ and $\lambda_h = M_h^2/M_W^2$. In addition, the first three arguments of the Passarino-Veltman loop functions, C_0 and C_{24}, are taken to be zero. In Eq. (14), B^R and C^L (assuming no left-right mixing) are mixing matrices parametrizing the couplings $W_R lN$ and ZNN, respectively. The value of U_{br}(R) depends on many kinematic variables, *i.e.*, the masses of heavy neutrinos (in our estimates we use $m_N =$ 4 TeV), the W_R-boson mass (M_R), and the charged Higgs mass (M_h).[21] Quantum effects of the remaining Higgs scalars are found to be rather small[21] — see also discussion in Section 6. In our numerical estimates, we use the typical values $(s_L^{\nu_\tau})^2 =$ 0.05 and $(s_L^{\nu_e})^2 = 0.01$. From Table 1, one can remark the complementary rôle that U_{br} and ΔA play to constrain or establish new-physics effects. Even if $U_{br}^{(\tau e)}$ could be unobservably small of order 10^{-3} for some range of kinematic variables, $\Delta\mathcal{A}_{\tau e}$ can be as large as 10% and hence capable of further constraining the parameter space of the model.

5. The minimal SUSY model

In this model,[22] the FCNC decay of the Z boson was estimated to be rather small, having $B(Z \to l_1 \bar{l}_2) < 1.\ 10^{-8}$.[23] In addition, the SUSY-SM can generate nonvanishing values for $U_{br}^{(ll')}(\mathrm{L})$ and $U_{br}^{(ll')}(\mathrm{R})$. These observables can be induced by left-handed and right-handed scalar leptons (denoted as $\tilde{l}_L$, $\tilde{l}_R$) as well as scalar neutrinos. A non-zero non-universal $Zl\bar{l}$ coupling can be produced if two non-degenerate left-handed or right-handed scalar leptons, say $\tilde{l}$ and $\tilde{l}'$, are present. To get a feeling about the size of the effects expected in this model, we will consider the SUSY limit of the gaugino sector, where only explicit SUSY-breaking scalar-lepton mass terms are taken into account. Then, only two neutralinos, the photino $\tilde{\gamma}$ and the "ziggsino" $\tilde{\zeta}$ with mass $m_{\tilde{\zeta}} = M_Z$, will contribute as shown in Fig. 2. "Ziggsino" is a Dirac fermion composed from degenerate Majorana states of a zino $\tilde{z}$ (the SUSY partner of the Z boson) and one of the higgsino fields. For our illustrations, we will further assume that only one scalar lepton $\tilde{l}$ is relatively light whereas the others, *e.g.* $\tilde{l}'$, are much heavier than M_Z. Due to the decoupling behaviour of softly broken SUSY theories, we can neglect quantum effects of $\tilde{l}'$. It is then straightforward to obtain [10]

$$
\begin{aligned}
U_{br}^{(ll')}(\mathrm{L}) &= -\frac{\alpha_w}{8\pi}\frac{g_L^4\cos 2\theta_L}{g_L^2+g_R^2}\left[\frac{g_R^2}{g_L^2}\int_0^1\int_0^1 dxdy\ y\ \ln\left(1-\frac{\lambda_Z}{\lambda_{\tilde{l}_L}}yx(1-x)\right)\right. \\
&\quad \left. +\lambda_Z\int_0^1\int_0^1 dxdy\ y\ \ln\left(\frac{\lambda_{\tilde{l}_L}y+\lambda_Z[1-y-y^2x(1-x)]}{\lambda_{\tilde{l}_L}y+\lambda_Z(1-y)}\right)\right], \qquad (18)\\
U_{br}^{(ll')}(\mathrm{R}) &= -\frac{\alpha_w}{8\pi}\frac{g_R^4\cos 2\theta_R}{g_L^2+g_R^2}\left[\int_0^1\int_0^1 dxdy\ y\ \ln\left(1-\frac{\lambda_Z}{\lambda_{\tilde{l}_R}}yx(1-x)\right)\right. \\
&\quad \left. +\lambda_Z\int_0^1\int_0^1 dxdy\ y\ \ln\left(\frac{\lambda_{\tilde{l}_R}y+\lambda_Z[1-y-y^2x(1-x)]}{\lambda_{\tilde{l}_R}y+\lambda_Z(1-y)}\right)\right], \qquad (19)
\end{aligned}
$$

where $\lambda_Z = M_Z^2/M_W^2$, $\lambda_{\tilde{l}_L} = m_{\tilde{l}_L}^2/M_W^2$, $\lambda_{\tilde{l}_R} = m_{\tilde{l}_R}^2/M_W^2$, and θ_L (θ_R) is a mixing angle between the two left-handed (right-handed) scalar leptons $\tilde{l}_L$ ($\tilde{l}_R$) and $\tilde{l}'_L$ ($\tilde{l}'_R$). From Table 1, we see that the universality-violating observables U_{br} and $\Delta\mathcal{A}$ turn out to be no much bigger than 10^{-3}. Nevertheless, in other SUSY extensions, the situation may be different. For instance, in SUSY models with right-handed neutrinos, enhancements coming from the SUSY Yukawa sector are expected to enter via the coupling of the charged higgsinos to leptons and scalar neutrinos. In such scenarios,

$\Delta\mathcal{A}_{\tau e}$ could then reach an experimentally accessible level $\sim 10^{-2}$.

6. The observable R_b

Another observable which will still be of interest is

$$R_b \;=\; 0.2202 \pm 0.0020 \qquad (\mathrm{SM} : 0.2158). \tag{20}$$

Assuming that the LEP measurement is correct, R_b turns out to be about 2σ off from the theoretical prediction of the minimal SM. New physics contributions to R_b can be conveniently calculated through [24]

$$R_b \;=\; 0.22\Big[1 + 0.78\nabla_b^{(SM)}(m_t) - 0.06\Delta\rho^{(SM)}(m_t)\Big], \tag{21}$$

where $\nabla_b^{(SM)}(m_t)$ and $\Delta\rho^{(SM)}(m_t)$ contains the m_t-dependent parts of the vertex and oblique corrections, respectively. Practically, only $\nabla_b^{(SM)}(m_t)$ gives significant negative contributions to R_b, which behave, in the large top-mass limit, as [25]

$$\nabla_b^{(SM)}(m_t) \;\simeq\; -\frac{20\alpha_w s_w^2}{13\pi}\,\frac{m_t^2}{M_Z^2}. \tag{22}$$

If there are new physics effects contributing to $\nabla_b^{(SM)}(m_t)$, these can be calculated by

$$\nabla_b^{(new)}(m_t) \;=\; \frac{\alpha_w}{2\pi}\,\frac{g_L^b\Re e(\Gamma_{bb}^L(m_t) - \Gamma_{bb}^L(0)) + g_R^b\Re e(\Gamma_{bb}^R(m_t) - \Gamma_{bb}^R(0))}{g_L^{b2} \;+\; g_R^{b2}}\,, \tag{23}$$

where $g_L^b = 1 - 2s_w^2/3$ and $g_R^b = -2s_w^2/3$.

In the following, we will try to address the question whether there exist possibilities of producing positive contributions to R_b within the SUSY-SM and LRSM. As has already been noticed in Section 3, only positive contributions to R_b are of potential interest, which will help to achieve a better agreement between theoretical prediction and the experimental value of R_b.

In the SUSY-SM, R_b can in principle receive positive contributions from the large Yukawa coupling of the charged higgsino to the scalar top quark and b quark.[26] Also, R_b can get enhanced from large $\tan\beta$ scenarios. However, considering a number of constraints originating from $B(b \to s\gamma)$,[27] relic abundances of the lightest SUSY particle,[28] the net SUSY effect on R_b is considerably reduced and R_b is found to be 0.2166, which is about 1.5σ below the experimental value given in Eq. (20).[29]

In LRSM, we first consider the Feynman graphs of Figs. 1(m) and 1(n), where the external leptons are replaced by b-quarks and virtual down-type quarks are running in the place of charged leptons. The interaction of the FCNC scalars ϕ_2^{0r} and ϕ_2^{0i} with the $d,\ s,\ b$ quarks is enhanced, since the corresponding couplings are proportional to the top-quark mass. In fact, the FCNC scalars generate effective $Zb\bar{b}$ couplings

of both V–A and V+A nature. In the limit $M_{\phi_2^{0r}},\ M_{\phi_2^{0i}} \gg M_Z$, the effective $Zb\bar{b}$ couplings take the simple form

$$\Re e(\Gamma^R_{bb}(m_t) - \Gamma^R_{bb}(0)) \;=\; \frac{1}{8}|V^R_{tb}|^2 \frac{m_t^2}{M_W^2}\left(\frac{\lambda_S+\lambda_I}{2(\lambda_S-\lambda_I)}\ln\frac{\lambda_S}{\lambda_I}\;-\;1\right), \tag{24}$$

$$\Re e(\Gamma^L_{bb}(m_t) - \Gamma^L_{bb}(0)) \;=\; -\frac{1}{8}|V^L_{tb}|^2 \frac{m_t^2}{M_W^2}\left(\frac{\lambda_S+\lambda_I}{2(\lambda_S-\lambda_I)}\ln\frac{\lambda_S}{\lambda_I}\;-\;1\right), \tag{25}$$

where $\lambda_S = M^2_{\phi_2^{0r}}/M_W^2$ and $\lambda_I = M^2_{\phi_2^{0i}}/M_W^2$. The analytic function in the parentheses of the r.h.s. of Eqs. (24) and (25) is always positive and equals zero when the two scalars ϕ_2^{0r}, ϕ_2^{0i} are degenerate. Substituting Eqs. (24) and (25) into Eq. (23), one easily finds that the SM value of R_b is further decreased. This may lead to the mass restriction

$$M_{\phi_2^{0r}} \;\simeq\; M_{\phi_2^{0i}}. \tag{26}$$

The mass relation (26) has been used in our numerical estimates. Other quantum corrections that could help to produce positive contributions to R_b are due to diagrams similar to Figs. 1(h) and 1(d). Indeed, an analogous calculation gives

$$\Re e(\Gamma^R_{bb}(m_t) - \Gamma^R_{bb}(0)) \;=\; -\frac{1}{4}|V^R_{tb}|^2 \frac{m_t^2}{M_W^2}\, s_\beta^2 c_\beta^2 \left(\frac{\lambda_h+\lambda_R}{2(\lambda_h-\lambda_R)}\ln\frac{\lambda_h}{\lambda_R}\;-\;1\right). \tag{27}$$

However, the l.h.s. of Eq. (27) is proportional to $s_\beta^2 = M_W^2/M_R^2$ yielding a rather small positive effect. The latter simply demonstrates the difficulty of radiatively inducing positive contributions to R_b within the LRSM.

7. Conclusions

We have found that lepton-flavour-violating Z-boson decays, lepton universality in the decays $Z \to l\bar{l}$, and universality of leptonic asymmetries form a set of complementary observables, so as to impose interesting limitations on model-building in the leptonic sector. To precisely demonstrate this, we have analyzed conceivable low-energy scenarios of unified theories, such as the SM with neutral isosinglets, the left-right symmetric model, and the minimal SUSY model. In particular, LRSMs can induce sizeable values for $\Delta\mathcal{A}_{\tau e}$ at the experimental visible level of $5-10\%$, whereas the observable U_{br} measuring deviations from universality in the leptonic partial widths of the Z boson may turn out to be rather small. As can also be seen from Table 1, the sign of $\Delta\mathcal{A}_{\tau e}$ could help to discriminate among the various theoretical scenarios beyond the SM. Finally, we have seen that appears rather difficult to obtain positive contributions to R_b in GUT-motivated scenarios. For example, in LRSMs, δR_b^{new} always tends to be negative, which favours FCNC scalars that are degenerate in mass. If the LEP measurement is indeed correct, this may point towards

supersymmetric physics of an underlying theory.

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